shape representation, curve interpolation, contour reconstruction, shape coefficients, Hurwitz-Radon matrices

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APPLICATION OF HURWITZ – RADON MATRICES IN SHAPE REPRESENTATION

Abstract

Computer vision needs suitable methods of shape representation and contour reconstruction. One of them, invented by the author and called method of Hurwitz-Radon Matrices (MHR), can be used in representation and reconstruction of shapes of the objects in the plane. Proposed method is based on a family of Hurwitz-Radon (HR) matrices. The matrices are skew-symmetric and possess columns composed of orthogonal vectors. Shape is represented by the set of nodes. It is shown how to create the orthogonal and discrete OHR operator and how to use it in a process of shape representation and reconstruction. MHR method is interpolating the curve point by point without using any formula or function.

1. INTRODUCTION

Significant problem in machine vision and computer vision [1] is that of appropriate shape representation and reconstruction. Classical discussion about shape representation is based on the problem: contour versus skeleton. This paper is voting for contour which forms boundary of the object. Contour of the object, represented by contour points, consists of information which allows us to describe many important features of the object as shape coefficients [2]. In the paper contour is dealing with a set of curves. Curve modeling and generation is a basic subject in many branches of industry and computer science, for example in the CAD/CAM software.

The representation of shape can have a great impact on the accuracy and effectiveness of object recognition [3]. In the literature, shape has been represented by many options including curves [4], graph-based algorithms and medial axis [5] to enable shape-based object recognition. Digital curve (open or closed) can be represented by chain code (Freeman's code). Chain code depends on selection of the started point and transformations of the object. So Freeman's code is one of the method how to describe and to find contour of the object. Analog (continuous) version of Freeman's code is the curve α - *s*. Another contour representation and reconstruction is based on Fourier coefficients calculated in Discrete Fourier Transformation

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(DFT). These coefficients are used to fix similarity of the contours with different sizes or directions. If we assume that contour is built from segments of a line and fragments of circles or ellipses, Hough transformation is applied to detect contour lines. Also geometrical moments of the object are used during the process of object shape representation [6]. Proposed MHR method requires to detect specific points of the object contour, for example in compression and reconstruction of monochromatic medical images [7]. Contour is also applied in shape decomposition [8]. Many branches of medicine, for example computed tomography [9], need suitable and accurate methods of contour reconstruction [10]. Also industry and manufacturing are looking for methods connected with geometry of the contour [11]. So suitable shape representation and precise reconstruction or interpolation [12] of object contour is a key factor in many applications of computer vision.

2. Contour Points Based Shape Representation

Shape can be represented by object contour, i.e. curves that create each part of the contour. One curve is described by the set of nodes $(x_i,y_i) \in \boldsymbol{R}^2$ (contour points) as follows in proposed method:

1. nodes (interpolation points) are settled at local extrema (maximum or minimum) of one of coordinates and at least one point between two successive local extrema;
2. each node (x_i,y_i) is monotonic in coordinates x_i or y_i (for example equidistance in one of coordinates);
3. one curve (one part of the contour) is represented by at least five contour points.

Condition 1 is done for the most appropriate description of a curve. So we have n curves C_1, C_2, ... C_n that build whole contour and each curve is represented by nodes according to assumptions 1-3.

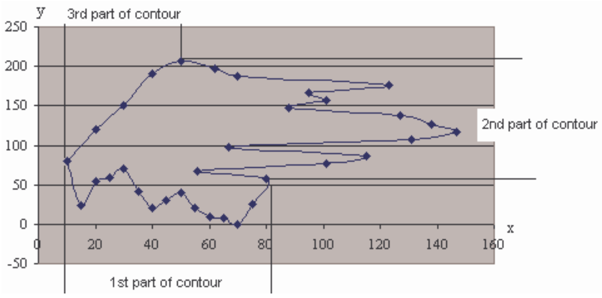

Fig. 1. Contour consists of three parts (three curves and their nodes)

Fig.1 is an example for $n = 3$: first part of contour C_1 is represented by nodes monotonic in coordinates x_i, second part of contour C_2 is represented by nodes monotonic in coordinates y_i and third part C_3 could be represented by nodes either monotonic in coordinates x_i or monotonic in coordinates y_i. Number of curves is optional and number of nodes for each curve is optional too (but at least five nodes for one curve). Representation points are treated as interpolation nodes. How accurate can we reconstruct whole contour using representation points? The shape reconstruction is possible using novel MHR method.

3. SHAPE RECONSTRUCTION VIA MHR METHOD

The following question is important in mathematics and computer sciences: is it possible to find a method of curve interpolation in the plane without building the interpolation polynomials and without mathematical form of the curve? Our paper aims at giving the positive answer to this question. There exists several well established methods: spline functions [13], shape-preserving techniques [14], subdivision algorithms [15], Bezier curves, B-splines, NURBS [16] and others [12] to overcome difficulties of polynomial interpolation, but matrix interpolation MHR (based on simple matrix calculations with low computational costs) seems to be quite novel in the area of shape reconstruction. In comparison MHR method with Bézier curves, Hermite curves and B-curves (*B-splines*) or NURBS one unpleasant feature of these curves must be mentioned: small change of one characteristic point can make big change of whole reconstructed curve [17]. Such a feature does not appear in MHR method [18]. Methods of curve interpolation based on classical polynomial interpolation: Newton, Lagrange or Hermite polynomials and spline curves which are piecewise polynomials [19]. Classical methods are useless to interpolate the function that fails to be differentiable at one point, for example the absolute value function $f(x) = |x|$ at $x = 0$. If point (0;0) is one of the interpolation nodes, then precise polynomial interpolation of the absolute value function is impossible. Also when the graph of interpolated function differs from the shape of polynomials considerably, for example $f(x) = 1/x$, interpolation is very hard because of existing local extrema of polynomial. Lagrange interpolation polynomial for function $f(x) = 1/x$ and nodes (5;0.2), (5/3;0.6), (1;1), (5/7;1.4), (5/9;1.8) has one minimum and two roots.

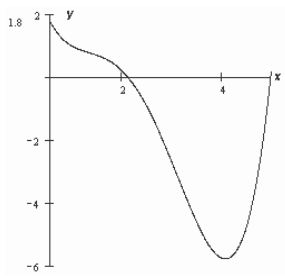

Fig. 2. Lagrange interpolation polynomial for nodes (5;0.2), (5/3;0.6), (1;1), (5/7;1.4), (5/9;1.8) differs extremely from the shape of function $f(x) = 1/x$

We cannot forget about the Runge's phenomenon: when interpolation nodes are equidistance then high-order polynomial oscillates toward the end of the interval, for example close to -1 and 1 with function $f(x) = 1/(1+25x^2)$ [20] or $f(x) = 1/(1+5x^2)$. Method of Hurwitz – Radon Matrices (MHR), described in this paper, is free of these bad features. Complexity of calculations for one unknown point in Lagrange or Newton interpolation based on n nodes is connected with the computational cost of rank $O(n^2)$. Complexity of calculations for L unknown points in MHR interpolation based on n nodes is connected with the computational cost of rank $O(L)$ [18]. This is very important feature of MHR method. The curve or function in MHR method is parameterized for value $\alpha \in [0;1]$ in the range of two or more successive interpolation nodes.

3.1. The Operator of Hurwitz – Radon

Adolf Hurwitz (1859-1919) and Johann Radon (1887-1956) published the papers about specific class of matrices in 1923, working on the problem of quadratic forms. Matrices A_i, $i = 1,2\ldots m$ satisfying

$$A_jA_k+A_kA_j = 0, \quad A_j^2 = -I \quad \text{for} \quad j \neq k;\ j, k = 1,2...m$$

are called *a family of Hurwitz - Radon matrices*. A family of Hurwitz - Radon (HR) matrices has important features [21]: HR matrices are skew-symmetric ($A_i^T = - A_i$) and reverse matrices are easy to find ($A_i^{-1} = - A_i$). Only for dimension N = 2, 4 or 8 the family of HR matrices consists of N - 1 matrices. For N = 2 there is one matrix:

$$A_1 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}.$$

For N = 4 there are three HR matrices with integer entries:

$$A_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix}, \quad A_3 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix}.$$

For N = 8 we have seven HR matrices with elements 0, ±1 [7]. So far HR matrices are applied in electronics [22]: in Space-Time Block Coding (STBC) and orthogonal design [23], also in signal processing [24] and Hamiltonian Neural Nets [25].

If one curve is described by a set of representation points $\{(x_i,y_i),\ i = 1, 2, \ldots, n\}$ monotonic in coordinates x_i, then HR matrices combined with identity matrix I_N are used to build an orthogonal and discrete Hurwitz - Radon Operator (OHR). For nodes (x_1,y_1), (x_2,y_2) OHR M of dimension N = 2 is constructed [26]:

$$B = (x_1 \cdot I_2 + x_2 \cdot A_1)(y_1 \cdot I_2 - y_2 \cdot A_1) = \begin{bmatrix} x_1 & x_2 \\ -x_2 & x_1 \end{bmatrix}\begin{bmatrix} y_1 & -y_2 \\ y_2 & y_1 \end{bmatrix}, \quad M = \frac{1}{x_1^2 + x_2^2} B,$$

$$M = \frac{1}{x_1^2 + x_2^2}\begin{bmatrix} x_1y_1 + x_2y_2 & x_2y_1 - x_1y_2 \\ x_1y_2 - x_2y_1 & x_1y_1 + x_2y_2 \end{bmatrix}. \tag{1}$$

Matrix M in (1) is found as a solution of equation:

$$\begin{bmatrix} a & b \\ -b & a \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}. \tag{2}$$

For nodes (x_1,y_1), (x_2,y_2), (x_3,y_3), (x_4,y_4), monotonic in x_i, OHR of dimension N = 4 is constructed [26]:

$$M = \frac{1}{x_1^2 + x_2^2 + x_3^2 + x_4^2}\begin{bmatrix} u_0 & u_1 & u_2 & u_3 \\ -u_1 & u_0 & -u_3 & u_2 \\ -u_2 & u_3 & u_0 & -u_1 \\ -u_3 & -u_2 & u_1 & u_0 \end{bmatrix} \tag{3}$$

where $u_0 = x_1y_1 + x_2y_2 + x_3y_3 + x_4y_4$, $u_1 = -x_1y_2 + x_2y_1 + x_3y_4 - x_4y_3$,
$u_2 = -x_1y_3 - x_2y_4 + x_3y_1 + x_4y_2$, $u_3 = -x_1y_4 + x_2y_3 - x_3y_2 + x_4y_1$.

Matrix M in (3) is found as a solution of equation:

$$\begin{bmatrix} a & b & c & d \\ -b & a & -d & c \\ -c & d & a & -b \\ -d & -c & b & a \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix}. \tag{4}$$

For nodes (x_1,y_1), (x_2,y_2), …, (x_8,y_8), monotonic in x_i, OHR of dimension $N = 8$ is built [26] similarly as (1) or (3). Note that OHR operators M (1)-(3) satisfy the condition of interpolation

$$M \cdot \mathbf{x} = \mathbf{y} \tag{5}$$

for $\mathbf{x} = (x_1,x_2\ldots,x_N)^T \in \boldsymbol{R}^N$, $\mathbf{x} \neq \mathbf{0}$, $\mathbf{y} = (y_1,y_2\ldots,y_N)^T \in \boldsymbol{R}^N$, $N = 2$, 4 or 8.

If one curve is described by a set of nodes $\{(x_i,y_i), i = 1, 2, \ldots, n\}$ monotonic in coordinates y_i, then HR matrices combined with identity matrix I_N are used to build an orthogonal and discrete reverse Hurwitz - Radon Operator (reverse OHR) M^{-1}. If matrix M in (1)-(3) has form:

$$M = \frac{1}{\sum_{i=1}^{N} x_i^2}(u_0 \cdot I_N + D),$$

where matrix D with elements $u_1, \ldots, u_{N-1}$ and zero diagonal, then reverse OHR M^{-1} is given by:

$$M^{-1} = \frac{1}{\sum_{i=1}^{N} y_i^2}(u_0 \cdot I_N - D). \tag{6}$$

Note that reverse OHR operator (6) satisfies the condition of interpolation

$$M^{-1} \cdot \mathbf{y} = \mathbf{x} \tag{7}$$

for $\mathbf{x} = (x_1,x_2\ldots,x_N)^T \in \boldsymbol{R}^N$, $\mathbf{y} = (y_1,y_2\ldots,y_N)^T \in \boldsymbol{R}^N$, $\mathbf{y} \neq \mathbf{0}$, $N = 2$, 4 or 8.

3.2. MHR method (basic version)

Key question looks as follows: how compute coordinates of points settled between interpolation nodes? A set of nodes is the only information about curve in basic version of MHR method. On a segment of a line every number “c” situated between “a” and “b” is described by a linear (convex) combination $c = \alpha \cdot a + (1 - \alpha) \cdot b$ for

$$\alpha = \frac{b-c}{b-a} \in [0;1]. \tag{8}$$

When the nodes are monotonic in coordinates x_i, average OHR operator M_2 of dimension N = 2, 4 or 8 is constructed as follows [7,26]:

$$M_2 = \alpha \cdot M_0 + (1-\alpha) \cdot M_1 \tag{9}$$

with the operator M_0 built (1)-(3) by "odd" nodes $(x_1=a,y_1)$, (x_3,y_3), …, (x_{2N-1},y_{2N-1}) and M_1 built (1)-(3) by "even" nodes $(x_2=b,y_2)$, (x_4,y_4), …, (x_{2N},y_{2N}). Having the operator M_2 for coordinates $x_i < x_{i+1}$ it is possible to reconstruct the second coordinates of points (x,y) in terms of the vector C defined with

$$c_i = \alpha \cdot x_{2i-1} + (1-\alpha) \cdot x_{2i} \quad , \quad i = 1, 2, \ldots, N \tag{10}$$

as $C = [c_1, c_2, \ldots, c_N]^T$. The required formula is similar to (5):

$$Y(C) = M_2 \cdot C \tag{11}$$

in which components of vector $Y(C)$ give the second coordinate of the points (x,y) corresponding to the first coordinate, given in terms of components of the vector C. On the other hand, having the operator M_2^{-1} for coordinates $y_i < y_{i+1}$ it is possible to reconstruct the first coordinates of points (x,y) [7,26]:

$$M_2^{-1} = \alpha \cdot M_0^{-1} + (1-\alpha) \cdot M_1^{-1}, \quad c_i = \alpha \cdot y_{2i-1} + (1-\alpha) \cdot y_{2i} \, ,$$

$$X(C) = M_2^{-1} \cdot C \quad . \tag{12}$$

Contour of the object is constructed with several number of curves. Calculation of unknown coordinates for contour points using (8)-(12) is called by author the method of Hurwitz - Radon Matrices (MHR). Here is the application of basic MHR method for functions $f(x) = 2/x$ (five nodes equidistance in first coordinate: x = 0.4, 0.7, 1.0, 1.3, 1.6) and $f(x) = 1/(1+5x^2)$ with five nodes for x = -1, -0.5, 0, 0.5, 1.

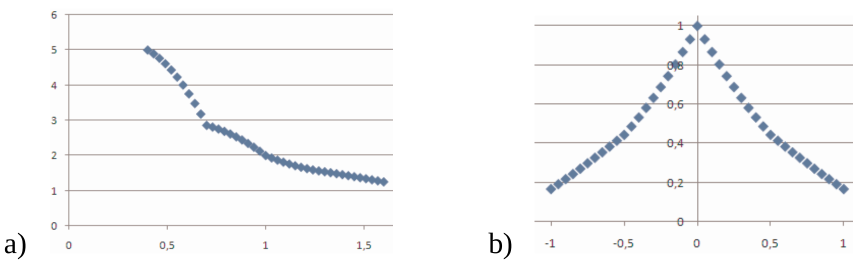

Fig. 3. Thirty six interpolated points of functions $f(x) = 2/x$ (a) and $f(x) = 1/(1+5x^2)$ (b) using basic MHR method with 5 nodes

Basic version of MHR method preserves monotonicity and symmetry (Fig.3b) of the graphs.

3.3. MHR method with parameter *k*

The curve $y = 2/x$ reconstructed by basic version of MHR method (Fig.3a) looks not quite accurate. For better reconstruction of the curve, appropriate $k \in (0;2]$ in MHR method with parameter k is calculated:

$$M_2 = \alpha^k \cdot M_0 + (1-\alpha^k) \cdot M_1 \tag{13}$$

or

$$M_2^{-1} = \alpha^k \cdot M_0^{-1} + (1-\alpha^k) \cdot M_1^{-1} . \tag{14}$$

For $k = 1$ MHR method (13-14) presents a basic version (9,12). In the case of $k > 2$ author's experiments confirm that models differ from the curves considerably. Choice of parameter k is connected with comparison of precise values w_i for function $f(x) = 2/x$ in control points p_i, situated in the middle between interpolation nodes ($\alpha = 0.5$):

$$p_i = \frac{1}{2}(x_i + x_{i+1}), \qquad w_i = f(p_i) = \frac{2}{p_i},$$

and values in control points p_i computed by MHR method. Control points are settled in the middle between interpolation nodes, because interpolation error of MHR method is the biggest [6]. Choice of rank k is done by criterion: difference between precise values w_i and values reconstructed by MHR method is the smallest. Control points p_i in this example are established for p_i = 0.55, 0.85, 1.15, 1.45. Four values of the curve are compared for various parameter $k \in (0;2]$. The best result is calculated for $k = 1.565$:

$$|w1 - 3.637| + |w3 - 1.624| + |w2 - 2.278| + |w4 - 1.322| = 0.248,$$

whereas basic version ($k = 1$) gives worse result:

$$|w1 - 4.23| + |w3 - 1.709| + |w2 - 2.532| + |w4 - 1.398| = 0.822 .$$

Reconstruction of the curve $y = 2/x$ by MHR method (13) with parameter $k = 1.565$ looks as follows:

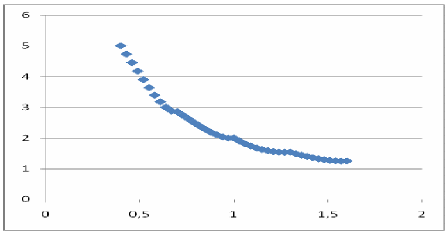

Fig. 4. The curve $y = 2/x$ modeled via MHR method for $k = 1.565$ and five nodes together with 36 reconstructed points

Fig.4 represents the curve $y = 2/x$ more precisely then Fig.3a. Convexity of reconstructed curve is very important factor in MHR method. Appropriate choice of parameter k is connected with regulation and controlling of convexity: model of the curve (Fig.4) preserves monotonicity and convexity.

3.4. MHR method for equidistance nodes

Assume that there is odd number of interpolation nodes (x_1,y_1), (x_2,y_2),…, (x_{2k+1},y_{2k+1}) in MHR method (k = 2,3,4…, k = *const.*) and all coordinates x_i or all coordinates y_i are equidistance (a fixed step of coordinates x_i or y_i). For example dealing with coordinate x_i we have the condition of proportion for first and second half of nodes (N = 2):

$$\forall i = 2,...,k : \frac{x_{k+1} - x_i}{x_{k+1} - x_1} = \frac{x_{2k+1} - x_{k+i}}{x_{2k+1} - x_{k+1}} = p_{i-1} . \tag{15}$$

Values $p_1 > ... > p_{k-1} \in (0;1)$ with $p_0 = 1$ and $p_k = 0$ are crucial in the process of interpolation. Let M_i (i = 0,1,2,…,k) is OHR operator of dimension N = 2 constructed (1) for nodes (x_{i+1},y_{i+1}) and (x_{k+i+1},y_{k+i+1}). Average OHR operator M_{k+1} is built as follows:

$$M_{k+1} = \sum_{i=0}^{k} s_i \cdot M_i . \tag{16}$$

Average OHR operator M_2 in basic version (9) is calculated as (16) for k = 1 and p_1 = 0.
Coefficients s_i are computed:

$$s_i = \frac{(\alpha - p_0)(\alpha - p_1)...(\alpha - p_{i-1})(\alpha - p_{i+1})...(\alpha - p_k)}{(p_i - p_0)(p_i - p_1)...(p_i - p_{i-1})(p_i - p_{i+1})...(p_i - p_k)} \quad , \tag{17}$$

$$s_i = \frac{\prod_{j=0,\, j\neq i}^{k} (\alpha - p_j)}{\prod_{j=0,\, j\neq i}^{k} (p_i - p_j)} , \qquad \sum_{i=0}^{k} s_i = 1$$

for any coordinate c_1 situated between x_1 and x_{k+1} (first half of nodes) as follows:

$$c_1 = \alpha \cdot x_1 + \beta \cdot x_{k+1} \quad \text{for} \quad 0 \leq \beta = 1 - \alpha \leq 1,$$

$$\alpha = \frac{x_{k+1} - c_1}{x_{k+1} - x_1} \in [0;1]. \tag{18}$$

Vector of second coordinates $Y(C) = [y(c_1), y(c_2)]^T$ is calculated:

$$Y(C) = M_{k+1} \cdot \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = M_{k+1} \cdot (\alpha \begin{bmatrix} x_1 \\ x_{k+1} \end{bmatrix} + (1-\alpha) \begin{bmatrix} x_{k+1} \\ x_{2k+1} \end{bmatrix}) . \tag{19}$$

Here is the example of average operator (16) for five nodes equidistance in coordinate x_i:
(x_1,y_1), (x_2,y_2),…, (x_5,y_5), k = 2, p_2 = 0, p_1 = ½, p_0 = 1.

$$M_0 = \frac{1}{x_1^2 + x_3^2}\begin{bmatrix} x_1y_1 + x_3y_3 & x_3y_1 - x_1y_3 \\ x_1y_3 - x_3y_1 & x_1y_1 + x_3y_3 \end{bmatrix}, \quad M_1 = \frac{1}{x_2^2 + x_4^2}\begin{bmatrix} x_2y_2 + x_4y_4 & x_4y_2 - x_2y_4 \\ x_2y_4 - x_4y_2 & x_2y_2 + x_4y_4 \end{bmatrix},$$

$$M_2 = \frac{1}{x_3^2 + x_5^2}\begin{bmatrix} x_3y_3 + x_5y_5 & x_5y_3 - x_3y_5 \\ x_3y_5 - x_5y_3 & x_3y_3 + x_5y_5 \end{bmatrix}.$$

$$s_0 = \frac{(\alpha-0)(\alpha-0.5)}{(1-0)(1-0.5)}, \quad s_1 = \frac{(\alpha-0)(\alpha-1)}{(0.5-0)(0.5-1)}, \quad s_2 = \frac{(\alpha-1)(\alpha-0.5)}{(0-1)(0-0.5)},$$

$$\sum_{i=0}^{2} s_i = 2\alpha(\alpha-\frac{1}{2}) - 4\alpha(\alpha-1) + 2(\alpha-1)(\alpha-\frac{1}{2}) = 1,$$

$$M_3 = 2\alpha(\alpha-\frac{1}{2})M_0 - 4\alpha(\alpha-1)M_1 + 2(\alpha-1)(\alpha-\frac{1}{2})M_2.$$

Here is the application of MHR method with equidistance nodes for function *f(x)*= 1/*x* and nine nodes equidistance in second coordinate: *y* = 0.2, 0.4, 0.6, 0.8, 1, 1.2, 1.4, 1.6, 1.8.

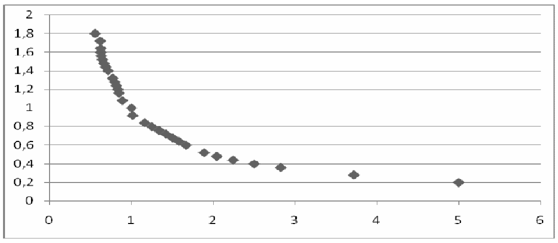

Fig. 5. Twenty two interpolated points of function *f(x)* = 1/*x* using MHR method with 9 equidistance nodes

MHR method for equidistance nodes requires the coefficients s_i (17) that are computed in similar way like Lagrange interpolation polynomial.

4. SHAPE COEFFICIENTS

Some of shape coefficients in object recognition are calculated using area of the object *S* and length of the contour *L*. For example:

$$R_S = \frac{L^2}{4\pi S}, \quad R_{C1} = 2\sqrt{\frac{S}{\pi}}, \quad R_{C2} = \frac{L}{\pi}, \quad R_M = \frac{L}{2\sqrt{\pi S}} - 1.$$

Area *S* is also applied in coefficients of: Blair-Bliss, Danielsson, compactness [27].

The contour is divided into *n* curves C_1, C_2, ... C_n. Having nodes (x_1,y_1), (x_2,y_2),…, (x_m,y_m) for each C_i in MHR method, it is possible to compute as many curve points as we want for any parameter $\alpha \in [0;1]$. Assume that *k* is the number of reconstructed points together with *m* nodes.

So curve C_i consists of k points which are indexed (x_1', y_1'), (x_2', y_2'),…, (x_k', y_k'), where $(x_1', y_1') = (x_1, y_1)$ and $(x_k', y_k') = (x_m, y_m)$. The length of curve C_i, consists of k points, is estimated:

$$d(C_i) = \sum_{i=1}^{k-1} \sqrt{(x_{i+1}' - x_i')^2 + (y_{i+1}' - y_i')^2} \cdot \tag{20}$$

Length of whole contour L is computed:

$$L = d(C_1) + d(C_2) + \ldots + d(C_n). \tag{21}$$

For example precise length of curve in Fig.3a is 4.045 and length calculated via (20) is $d(C_1) = 4.050$. Precise length of curve in Fig.3b is 2.679 and length calculated via (20) is $d(C_2) = 2.643$.

Area of the object can be divided horizontally or vertically into the set of l polygons: triangles, squares, rectangles, trapezoids.

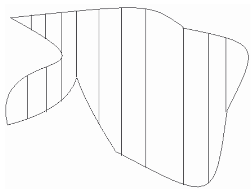

Fig. 6. The object area consists of polygons

Coordinates of corners for each polygon P_i are calculated by MHR method and then it is easy to estimate the area of P_i. For example P_1 as trapezoid (Fig.7) with corners (x_1, y_1), (x_1, y_2), (x_2, y_3), (x_2, y_4). Area of trapezoid P_1 is computed:

$$s(P_1) = \frac{1}{2} |x_2 - x_1| \cdot (|y_2 - y_1| + |y_4 - y_3|) \cdot \tag{22}$$

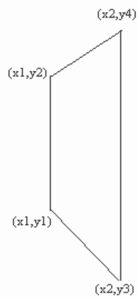

Fig. 7. Trapezoid as a part of the object.

Estimation of object area S is given by formula:

$$S = \sum_{i=1}^{l} s(P_i). \tag{23}$$

Contour points, calculated by MHR method [18], are applied in shape coefficients.

5. CONCLUSIONS

The method of Hurwitz-Radon Matrices leads to contour interpolation and shape reconstruction depending on the number and location of representation points. No characteristic features of curve are important in MHR method: failing to be differentiable at any point, the Runge's phenomenon or differences from the shape of polynomials. These features are very significant for classical polynomial interpolations. MHR method gives the possibility of reconstruction a curve consists of several parts, for example closed curve (contour). The only condition is to have a set of nodes for each part of a curve or contour according to assumptions in MHR method. Shape representation and curve reconstruction by MHR method is connected with possibility of changing the nodes coordinates and reconstruction of new curve or contour for new set of nodes, no matter what shape of curve or contour is to be reconstructed. Main features of MHR method are: accuracy of shape reconstruction depending on number of nodes and method of choosing nodes; reconstruction of curve consists of L points is connected with the computational cost of rank $O(L)$ [18]; MHR method preserves monotonicity and symmetry of the graphs, but convexity not always (selection of parameter k).

Future works are connected with: geometrical transformations of contour (translations, rotations, scaling)- only nodes are transformed and new curve (for example contour of the object) for new nodes is reconstructed, possibility to apply MHR method to three-dimensional curves [26] and connection MHR method with object recognition.

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